

Saturation in DIS at low x

D. Schildknecht^a * †

^aDepartment of Physics, University of Bielefeld,
P.O. Box 10 01 31, 33501 Bielefeld, Germany

Saturation at low x appears as an almost unavoidable consequence of the two-gluon exchange generic structure.

In this written version of my talk I will restrict myself to giving a brief discussion on the empirical evidence for the concept of “saturation” at low x in deep inelastic lepton-nucleon scattering.

In the model-independent analysis of the experimental data from HERA on DIS at low x carried out in the summer of the year 2000, we found[1] that the data on the total virtual photoabsorption cross section lie on a universal curve when plotted against the dimensionless variable

$$\eta = \frac{Q^2 + m_0^2}{\Lambda_{sat}^2(W^2)}, \quad (1)$$

where

$$\Lambda_{sat}^2(W^2) = B \left(\frac{W^2}{W_0^2} + 1 \right)^{C_2} \simeq B \left(\frac{W^2}{W_0^2} \right)^{C_2}. \quad (2)$$

Compare fig. 1. The energy-dependent quantity, $\Lambda_{sat}^2(W^2)$, acts as the scale (“saturation scale” or “saturation momentum”) that determines the range of Q^2 in which the energy dependence (at fixed Q^2) is either hard ($\eta \gg 1$) or soft ($\eta \ll 1$). The model-independent analysis only rest on the assumption that $\sigma_{\gamma^*p}(W^2, Q^2)$ be a smooth function of η . The fitting procedure gave[1,2]

$$\begin{aligned} m_0^2 &= 0.15 \pm 0.04 GeV^2, \\ W_0^2 &= 1081 \pm 12 GeV^2, \\ C_2 &= 0.27 \pm 0.01. \end{aligned} \quad (3)$$

As long as only smoothness of σ_{γ^*p} is assumed, the constant B can be arbitrary. With the explicit form of σ_{γ^*p} in the generalized vector

*Supported by DFG, contract Schi 189/6-2

†Presented at Diffraction 2004, Cala Gonone, Italy, September 18-23, 2004

dominance-color dipole picture (GVD-CDP), we found

$$B = 2.24 \pm 0.43 GeV^2. \quad (4)$$

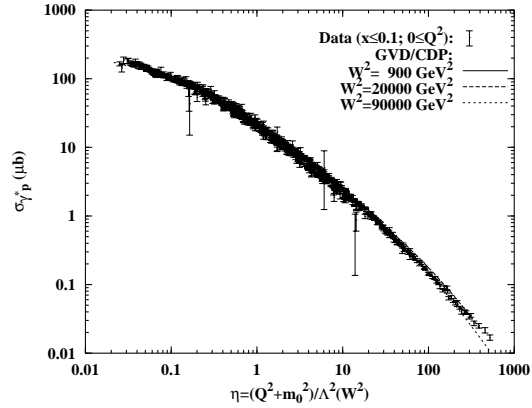


Figure 1. The total photoabsorption cross section as a function of the scaling variable η from (1).

Note that the data shown in fig. 1 include all data available for $x \simeq Q^2/W^2 < 0.1$ and $0 \leq Q^2 < 1000 GeV^2$, in particular, photoproduction ($Q^2 = 0$) is included.

Since the HERA energy, W , is limited, for large values of Q^2 small values of $\eta \ll 1$ cannot be explored. The low- η region in fig. 1 contains data close to photoproduction, while the large- η region is populated by large- Q^2 measurements. Nevertheless, fig. 1 suggests that the “saturation” property[1,2]

$$\lim_{\substack{W^2 \rightarrow \infty \\ Q^2 \text{ fixed}}} \frac{\sigma_{\gamma^*p}(\eta(W^2, Q^2))}{\sigma_{\gamma p}(W^2)} = 1 \quad (5)$$

to be valid for any fixed Q^2 .

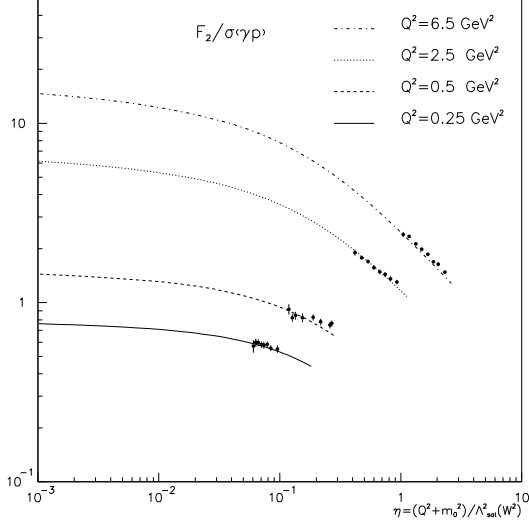


Figure 2. The ratio of the structure function $F_2(x, Q^2)$ and the photoabsorption cross section as a function of η .

In terms of the structure function

$$F_2(x, Q^2) \simeq \frac{Q^2}{4\pi^2\alpha} \sigma_{\gamma^* p}(\eta(W^2, Q^2)), \quad (6)$$

where $x \simeq Q^2/W^2$, according to (5) we have

$$\lim_{\substack{W^2 \rightarrow \infty \\ Q^2 \text{ fixed}}} 4\pi^2\alpha \frac{F_2(x, Q^2)}{\sigma_{\gamma p}(W^2)} = Q^2. \quad (7)$$

An explicit empirical test of the approach to saturation accordingly requires to plot the data for the ratio of the structure function $F_2(x, Q^2)$ and the photoproduction cross section as a function of η at fixed Q^2 . Saturation requires the ratio (7) to become flat and approach the value of Q^2 as a function of η as soon as η becomes small, $\eta \ll 1$.

The plot of the experimental data in fig. 2[3], for $Q^2 \lesssim 0.5 \text{ GeV}^2$ shows the expected flattening in the η -dependence for $\eta \ll 1$. For larger values of Q^2 the expected flattening for $\eta \lesssim 0.1$ cannot be verified at present due to lack of energy.

No explicit theoretical ansatz is needed for the plots in figs. 1 and 2. We have nevertheless included the theoretical curves from the GVD-CDP[1,2,4] that provides a theoretical basis for the observed scaling in η .³

³During discussion, I was asked which variable “from the

As conjectured[5,6] a long time ago, DIS at low x in terms of the virtual-photon-proton Compton amplitude is to be understood in terms of diffractive forward scattering of the hadronic $(q\bar{q})^{J=1}$ (vector) states the virtual photon dissociates or fluctuates into. With the advent of QCD, the underlying Pomeron exchange became understood in terms of the coupling of two gluons[8] to the $(q\bar{q})^{J=1}$ state. The gauge-theory structure implies that the $(q\bar{q})_{T,L}^{J=1} p$ color-dipole cross section, proportional to the imaginary part of the $(q\bar{q})_{T,L}^{J=1} p$ forward-scattering amplitude, takes the form[9,4]

$$\begin{aligned} \sigma_{(q\bar{q})_{T,L}^{J=1} p}(\vec{r}'_{\perp}, W^2) & \int d^2 \vec{l}'_{\perp} \bar{\sigma}_{(q\bar{q})_{T,L}^{J=1} p}(\vec{l}'_{\perp}, W^2) \cdot \\ & \cdot (1 - e^{-i\vec{l}'_{\perp} \vec{r}'_{\perp}}) \\ & \simeq \sigma^{(\infty)} \begin{cases} 1, & \text{for } \vec{r}'_{\perp}{}^2 \rightarrow \infty, \\ \frac{1}{4} \vec{r}'_{\perp}{}^2 \Lambda_{sat}^2(W^2), & \text{for } \vec{r}'_{\perp}{}^2 \rightarrow 0, \end{cases} \end{aligned} \quad (8)$$

where by definition

$$\sigma^{(\infty)} \equiv \pi \int d^2 \vec{l}'_{\perp} \bar{\sigma}_{(q\bar{q})_{T,L}^{J=1} p}(\vec{l}'_{\perp}, W^2), \quad (9)$$

and

$$\Lambda_{sat}^2(W^2) \equiv \frac{\pi}{\sigma^{(\infty)}} \int d^2 \vec{l}'_{\perp} \bar{\sigma}_{(q\bar{q})_{T,L}^{J=1} p}(\vec{l}'_{\perp}, W^2). \quad (10)$$

The (virtual) photoabsorption cross section is obtained from (8) by multiplication with the (light-cone) photon-wave function and subsequent integration over the transverse $q\bar{q}$ separation $\vec{r}'_{\perp} = \vec{r}'_{\perp}/\sqrt{z(1-z)}$ and the variable z with $0 \leq z \leq 1$ that e.g. determines angular distribution of the quark in the $q\bar{q}$ rest frame.

It is important to note that the two-gluon-exchange dynamical mechanism evaluated for $x \rightarrow 0$ implies the existence of the saturation scale $\Lambda_{sat}^2(W^2)$ according to (8). The scale $\Lambda_{sat}^2(W^2)$ is related to the effective value of the gluon transverse momentum, $\vec{l}'_{\perp} = \vec{l}'_{\perp}/\sqrt{z(1-z)}$, that enters the photoabsorption cross section as a consequence of the two-gluon-exchange mechanism.

point of view of the consumer should be bought”, η or $\tau \sim (x/x_0)^\lambda$ [7]. The answer: both, experiment is the arbiter. The choice of $\Lambda_{sat}^2(W^2)$ follows from the mass dispersion relation of generalized vector dominance and as such is well-motivated.

While an energy-independent scale $\Lambda_{sat}^2 = \text{const}$ a priori cannot be strictly excluded, it appears theoretically unlikely. Among other things, constancy would mean that the effective gluon transverse momentum from (10) would be energy independent, the diffractively produced $q\bar{q}$ mass spectrum be energy independent, the full W dependence reduced to a factorizing W dependence due to a potential (weak) energy dependence of $\sigma^{(\infty)}$ alone, etc. The generic two-gluon-exchange structure “almost” rules out $\Lambda_{sat}^2 = \text{const}$ and accordingly requires saturation.

Taking advantage of the fact that the ($J = 1$ part of the) dipole cross section (8) is essentially determined by the quantities $\sigma^{(\infty)}$ and $\Lambda_{sat}^2(W^2)$ in (9) and (10), the total photoabsorption cross section becomes approximately [1,2,4]

$$\sigma_{\gamma^*p}(W^2, Q^2) \simeq \frac{\alpha R_{e^+e^-}}{3\pi} \sigma^{(\infty)} \quad (11)$$

$$\begin{cases} \ln \frac{\Lambda_{sat}^2(W^2)}{Q^2 + m_0^2}, & (Q^2 \ll \Lambda_{sat}^2(W^2)), \\ \frac{1}{2} \frac{\Lambda_{sat}^2(W^2)}{Q^2} & (Q^2 \gg \Lambda_{sat}^2(W^2)). \end{cases}$$

A detailed evaluation leads to the theoretical results displayed in figs. 1 and 2.

Several remarks are appropriate:

- i) Unitarity for the hadronic ($q\bar{q}$) proton interaction requires the integral (9) to exist and $\sigma^{(\infty)}$ to be at most weakly dependent on the energy W . The fit yields $\sigma^{(\infty)} \simeq \text{const} \simeq 30 \text{mb}$.
- ii) The existence of a scale, $\Lambda_{sat}^2(W^2)$, according to (10) appears as a straightforward consequence of the two-gluon-exchange structure. This structure implies that the forward-scattering amplitude depends on the effective gluon transverse momentum.
- iii) Since unitarity ($\sigma^{(\infty)} \simeq \text{const}$) cannot be disputed, and the assumed two-gluon exchange generic structure seems safe, once $\Lambda_{sat}^2 = \text{const}$ is abandoned, we must have the transition to the logarithmic behavior in (11), i.e. saturation as depicted in fig. 2 even far beyond the energy range accessible at present.
- iv) The gluon structure function from (8) is given by [10] $\alpha_s(Q^2) x g(x, Q^2) = \frac{1}{8\pi^2} \sigma^{(\infty)} \Lambda_{sat}^2 \left(\frac{Q^2}{x} \right)$, again disfavoring constancy of $\Lambda_{sat}^2(W^2)$.
- v) When saturation and the logarithmic behav-

ior in (11) set in, the usual connection between F_2 and the gluon structure function breaks down. An extensive literature (compare e.g. [11] and the references therein) attempts to apply (nonlinear) evolution equations for gluon distributions even in this logarithmic domain.

ACKNOWLEDGEMENT

It is a pleasure to thank Masaaki Kuroda for a fruitful collaboration and Roberto Fiore and Alessandro Papa for organising a very successful meeting in splendid surroundings.

REFERENCES

1. D. Schildknecht, in Diffraction 2000, Nucl. Phys. (Proc. Suppl.) 99 (2001) 121; D. Schildknecht, B. Surrow, M. Tentyukov, Phys. Lett. B499 (2001) 116.
2. G. Cvetic, D. Schildknecht, B. Surrow, M. Tentyukov, Eur. Phys. J. C20 (2001) 77; D. Schildknecht, B. Surrow, M. Tentyukov, Mod. Phys. Lett. A Vol. 16 (2001) 1829; D. Schildknecht, in “The 9th International Workshop on Deep Inelastic Scattering DIS 2001”, Bologna, Italy, editors G. Bruni et al. (World Scientific 2002) 798.
3. M. Kuroda, D. Schildknecht, in preparation.
4. M. Kuroda, D. Schildknecht, Phys. Rev. D66 (2002) 094005.
5. J.J. Sakurai, D. Schildknecht, Phys. Lett. 40B (1972) 121; B. Gorczyca, D. Schildknecht, Phys. Lett. 47B (1973) 71
6. H. Fraas, B.J. Read, D. Schildknecht, Nucl. Phys. B86 (1975) 346; Nucl. Phys. B88 (1975) 301; R. Devenish, D. Schildknecht, Phys. Rev. D19 (1976) 93.
7. A.M. Stasto, K. Golec-Biernat, J. Kwiecinski, Phys. Rev. Lett. B86 (2001) 596.
8. F.E. Low, Phys. Rev. D12 (1975) 163; S. Nussinov, Phys. Rev. Lett. 34 (1975) 1286; Phys. Rev. D14 (1976) 246; J. Gunion, D. Soper, Phys. Rev. D15 (1977) 2617.
9. N.N. Nikolaev, B.G. Zakharov, Z. Phys. C49 (1991) 607; Z. Phys. C53 (1992) 331; Soviet Phys. JETP 78 (1994) 598.
10. N.N. Nikolaev, B.G. Zakharov, Phys. Lett. B332 (1994) 184.
11. E. Iancu, K. Itakura, S. Munier, hep-ph/0310338.